VOLATILITY MODELS

UNIVARIATE

1. ARCH
2. GARCH
3. GJR GARCH
4. APARCH
5. VaR.

MULTIVARIATE

1. VEC
   - Vech
   - Diagonal.
2. BEKK
3. CCC - model
4. DCC - model
5. BDCC → RS-DCC
6. stochastic volatility

$E(e_t) = 0$ — (3)

$e_t \sim N(0, \sigma^2)$ — (4)

$E(X-\mu)^2 = \sigma^2$ — (5)

$Cov\left[(X-\mu_i)(X-\mu_j)\right] = 0$
$i \neq j$

We want the variance to be constant over time

$\hat{y}_t = \alpha_1 + \alpha_2 X_{1t} + \cdots + \alpha_k X_{kt} + e_t$

y_t (T×1) vector

(T×1) vector

$t = 1$

$t = 1, \ldots, T$

$e = Y - \hat{Y}$

$\Sigma e^2 = \Sigma (y - \hat{y})^2$

(T×1) vector of errors

$$\begin{bmatrix} e_1 \\ \vdots \\ \vdots \\ e_n \end{bmatrix}$$

$e^2 = e \cdot e$

$= e \cdot e'$ (prime)

$$\begin{bmatrix} e_1 & e_2 & \ldots & e_n \end{bmatrix}$$

$$= \begin{bmatrix} e_1e_1 & e_1e_2 & \ldots & e_1e_n \\ e_2e_1 & e_2e_2 & \ldots & e_2e_n \\ \vdots & & & \\ e_ne_1 & e_ne_2 & \ldots & e_ne_n \end{bmatrix}$$

$$= \begin{bmatrix} e_1^2 & e_1e_2 & \ldots & e_{1n} \\ e_2e_1 & & & \\ \vdots & & & \\ e_ne_1 & e_ne_2 & \ldots & e_n^2 \end{bmatrix}$$

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$$E(e_t) = 0$$

$$Cov(e_i e_j) = 0$$

$$\begin{bmatrix} E(e_t^2) & \overset{0}{E(e_1 e_2)} & \cdots & \overset{0}{E(e_1 e_n)} \\ E(e_2 e_1) & E(e_2^2) & \cdots & \overset{0}{E(e_2 e_n)} \\ \vdots & & & \\ E(e_n e_1) & E(e_n e_2) & \cdots & E(e_n^2) \end{bmatrix} = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$\sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \leftarrow$$ Identity matrix of size n.

$$\Longrightarrow \sigma_i^2 I_n$$

$\sigma_1^2 I_n \quad e_2^2 I_n \quad e_3^2 I_n$

Engle (1982) came up with an Engle ARCH test

ARCH- LM

$\sigma^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + \cdot e_{t-2}^2 + \cdots + e_{t-q}^2$

$\sigma^2 = \gamma_0 + \sum_{i=1}^{N} \gamma_i e_{t-i}$

$H_0 : \gamma_i = 0$ (Homo)

$H_1 : \gamma_i \neq 0$ (Hetero)

① AR, ② MA, ③ ARMA, ④ ARMAX,

Endogenous

⑤ Frictional Integrated

ARFIMA

AR

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_n y_{t-p} + e_t$$

e_t IN.

(MA).

$$y_t = \alpha_0 + \alpha_1 e_t + \alpha_2 e_{t-1} + \dots + \alpha_k e_{t-q}$$

(ARMA)

$$y_t = \alpha_0 + \alpha_2 y_{t-1} + \dots \alpha_k y_{t-p} + \beta_0 e_t + \beta_1 e_{t-1} + \dots$$

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(ARMAX)

$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots \alpha_k y_{t-p} + \beta_0 e_t + \beta_1 e_{t-1} + \dots \gamma_1 X_t$

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MEAN EQ.

(a) VAR (b) VAM (c) VARMA — specification of mean of multivariate

VAR

$$y_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i Y_{t-i} + e_t$$

$e_t \sim (0, \sigma^2)$

$e_t \sim (0, \sigma_i^2)$ — This will not hold in case of returns.

$r_{1t} = \gamma_0 + \gamma_1 r_{1t} + \dots + \beta_1 r_{2t-1} + e_{r_1}$

$r_{2t} = \beta_0 + \beta_1 r_{2t-1} + \dots + \gamma_1 r_{1t-1} + e_{r_2}$

$$\begin{bmatrix} e_{r_1} & e_{r_2} \\ \vdots & \vdots \\ e_{r_n} & e_{r_m} \end{bmatrix} \begin{bmatrix} e_{r_1} \cdots & e_{r_n} \\ e_{r_1} \cdots & e_{m} \end{bmatrix} = \begin{bmatrix} \sigma^2_{r_1} & \sigma^2_{r_2} \\ & \end{bmatrix}$$

This is the case of conceptual difference.

VAR 1^{st} eq. $\sim N(0, \sigma^2_{r_1})$

VAR 2^{nd} eq. $\sim N(0, \sigma^2_{r_2})$

$$\begin{bmatrix} \sigma^2_{r_1} \\ \sigma^2_{r_2} \end{bmatrix} = \begin{bmatrix} \gamma_0 \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \gamma_1 & \beta_1 \\ \gamma_2 & \beta_2 \end{bmatrix} \begin{bmatrix} \epsilon^2_{r_1} \\ \epsilon^2_{r_2} \end{bmatrix}$$ — Multivariate Volatility model

b. Alternative to VAR model is VMA (Vector Moving Average)

$$r_t = [r_{1t}, r_{2t}]$$

$$r_t = \gamma_0 + \gamma_1 e_t + \gamma_2 e_{t-1} + \dots + \gamma_q e_{t-q}$$

γ_1: (2×2)

e_t: (2×1)

$e_t \sim (0, \sigma^2)$ Both the errors are i.i.d

$$r_t = [r_{1t-1}, r_{2t}]$$

current of e_t

$$r_t = r_0 + \sum_{i=1}^{p} \gamma_2 r_{t-1} + \sum_{j=1}^{q} \beta_j e_{t-j} + \beta_0 e_t$$

$e p_1 \sigma$, $e p_2 \sigma$

(VARMA (p, q))

Mean

AR- ARCH

MA- ARCH

ARMA- ARCH

ARMAX- ARCH

ARFIMA- ARCH

VARMA- BEKK

VARMA- GARCH

VARMA- CCC

VARMA- DCC

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GARCH- MEAN

$$y_t = \gamma_0 + \gamma_1 X_t + \gamma_2 + u_t$$

$$u_t \sim iid(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i h_{t-1} + \sum_{j=1}^{q} B_j e^2_{t-j}$$

GARCH ~~g~~- mean (p, q)

GARCH ~~g~~- mean is symmetric.

T- GARCH

Zakolan (1990); Glosten, Joganathan and Runkle (1993) - T GARCH

GJR- GARCH

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It considers the role of information.

It is empirically established that negative news creates larger volatility

$$d_t = \begin{cases} 1 \text{ if } u_t < 0 & \text{negative news} \\ 0 \text{ if } u_t > 0 & \text{positive news} \end{cases}$$

$$h_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i h_{t-i} + \sum_{j=1}^{q} (\beta_j + \theta_j d_{t-1}) u^2_{t-j}$$

should be significant

multiplicative dummy

θ_j : represent the asymmetric coefficient

If $\theta_j = 0$ symmetric mode

$\theta_j \neq 0$ asymmetric mode

$\theta_j > 0$

$\theta_j < 0$

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E – GARCH

Nelson (1991) — ① is able to cover the fat tail.

T- GARCH cannot capture the fat tail

$$h_t = \gamma_0 + (\gamma_1 e^2_{t-1} + \theta_1 d_{t-1}) + B_1 h_{t-1}.$$

By ensuring that $\log(h_t)$

$$\log(h_t) = \beta + \sum_{i=1}^{q} \left| \frac{u_{t-i}}{\sqrt{h_{t-j}}} \right| + \sum_{i=1}^{q} \lambda_i \frac{u_{t-i}}{\sqrt{h_{t-i}}} + \sum \delta_j \cdot \log(h_{t-j})$$

α_i = ARCH Coefficient

λ_i = E~~xp~~onential term

δ_j = GARCH term

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$\alpha_i + \delta_j$ = Volatility

$\lambda_1 = \lambda_2 = \ldots\ldots = \lambda_n = 0$ Symmetric model

$\lambda_1 \neq \lambda_2 \neq \ldots\ldots = \lambda_n = 0$

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If λ_2 is insignificant the model will be GARCH(1,1)

$$GARCH(1,1) = ARCH(q)$$

* The $\lambda_1 = \dots \lambda_n = 0$ is about significant or not being significa

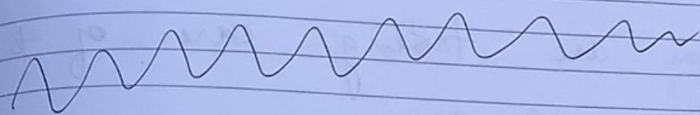
I-GARCH (Integrated)

Danidson (2004)

In many instance the sum of α and β is equal to 1.

0	low volatility	Impact of shock will have short memory on volatility
1<	high volatility	
=1	persistence	
>1	distortion	

The volatility can be persistent



The idea is to constrain the parameters so that

$$0 < \alpha \le 1$$

$$h_t = \gamma_0 + \sum_{i=1}^{q} \alpha_i e_{t-i} + \sum_{j=1}^{p} (1-\alpha_j) h_{t-1}$$

IGARCH(p, q)

$i = j$

IGARCH(1,1)

$$h_t = \gamma_0 + \alpha_1 e^2_{t-1} + (1-\alpha_1) h_{t-1}$$

$$h_t = \gamma_0 + \alpha_1 e^2_{t-1} + h_{t-1} - \alpha_1 h_{t-1}$$

$h_t = \gamma_0 + h_{t-1} + \alpha_1 \left(e^2_{t-1} - h_{t-1} \right)$

* you are taking care of the first 2 cases
* if it satisfies the equality then we will have to take the second difference

This model ensure that

$0 < \alpha + \beta < 1$

This model is only taking into consideration the low volatility and high volatility and not none.

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APARCH MODEL

DING, GRANGER AND ENGLE (1993) - MODEL.

APARCH(p, q)

$$\sigma_t^{\delta} = \gamma + \sum_{i=1}^{q} \alpha_i (|e_{t-i}| - \theta_i e_{t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta}$$

for $\delta = 2$ $\quad 0 < \theta_i < 1 \quad (i = 1 \ldots q)$

$\gamma = 0$

The stationary condition holds when $\gamma > 0$

$$\sum_{i=1}^{q} \alpha_i (|z| - \theta_i z) + \sum_{j=1}^{p} \beta_j < 1$$

$z \sim N(0,1)$

Symmetric model.

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Engle (1982) if this holds in the previous equation

$\delta = 2$

$\theta_i = 0 \; (i = 1, \ldots, q)$

$\beta_j = 0 \; (j = 1 \ldots, p)$

Bollerser model (1986)

if $\delta = 2$

$\theta_i = 0 \; (i = 1 \ldots p)$

Taylor (1986) or Schwert (1990) version if $\delta = 1$

$\theta_i = 0 \; (i = 1, \ldots, q)$

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GJR-GARCH (1993)

This holds when $\delta = 2$

Zakoion (1994) – TGARCH

This holds when $\delta = 1$

Asymmetric model

– Higgins and Bera (1992)

NARCH

This holds if

$\theta_i = 0 \; (i \ldots q)$

$\beta_j = 0 \; (j \ldots p)$

Geweke (1986) / Pentula (1986)

log-GARCH if $\delta \to 0$

→ Same as E-GARCH

If $\delta = 2$ the model collapses to Nelson (1991)

If any of ~~the~~ above 8 model fails then the IGARCH will be imposed because the shock takes long time to die off

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J.P. MORGAN EWMA MODEL

FORECAST VARIANCE AND COVARIANCE

$$\sigma_t^2(EWMA) = (1-\lambda)\, e_{t-1}\, e'_{t-1} + \lambda\, \sigma^2_{t-1} \quad [EWMA(1,1)]$$

λ = decay parameter which measures the impacts of shocks (news or noise) on volatility.

The empirical evidence is that this model collapses to IGARCH model.

The range of weight is $0 < \lambda \leq 1$

RISK METRICS

It was introduced to ensure stationarity by assuming that the decay parameter λ is equals to 0.94 for daily data

This constrains the model

λ is 0.97 for monthly data

VALUE AT RISK (VaR)

Used for quantifying risk.

- commodity risk.
- security risk
- market risk (financial and credit risk)
- corporate risk

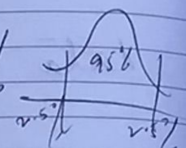
worst expected loss a firm can endure over a given period of time under normal market condition at a certain confidence interval

$VaR = 0.01$

nominal = Rs. 1000

Return = 10%

5%



$= 0.01 \times 1000 = 10$

It includes inflation risk, exchange rate fluctuation, and other unanticipated risk.

There are 3 methods

1) Variance Covariance
2) Historical Simulation
3) Monte Carlo Simulation

INDIVIDUAL ASSET (EX.)

Nominal asset = Rs. 10.

Price = Rs. 100

Average return = 7.35%

S.D/Volatility = 1.99%

Average return = $\frac{7.35}{100} \times 10,000,000$

= 7,35,000

Volatility can be calculated in

1) Historical volatility
2) Implied Volatility

C.I = 5%

VaR index = Volatility × level of significance

t is selected
distribution

$= 0.0199 \times 1.654$

$= 0.0329.$

$= 3,29,149.$

Risk lover will take 1% C.I

Risk neutral will take 5% C.I.

Risk averse will take 80% C.I.

INDIVIDU.

Nominal

Price = Rs

Average re

PORTFOLIO EXAMPLE (2)

Portfolio [X, Y]

$V_{portfolio} = \sqrt{x^2+y^2+2xy\,\rho(xy)}$

$\rho(xy)$

x^2 = Variance of asset X

y^2 = Variance of asset Y

ρ = correlation / covariance

$$C.V = \frac{\sum(x-\bar{x})(y-\bar{y})}{n}$$

$$r = \frac{Cov(x, y)}{S_x \cdot S_y}$$

VaR = Volatility of Index portfolio X Standardised value of α.

(different ways of calculating portfolio volatility)

* CaR = Cost of Risk Model.

* CFaR = Cash Flow at Risk Model