FA-⊗-VAR

DATE 09/05/2016

VAR
① overparameterization
② lack of economic meaning
③ Limited Information

SVAR
① and ②

-① and ③
FA-S-VAR

Literature of Price puzzle

Sims came up with composite price index

Augumenting Factor is very important

↓

we have to introduce Factor Modelling Approach

Factor Analysis is used to reduce data dimension

$$X_t = (x_{it}, \ldots, x_{Nt})$$

$N = 20$

$i = 1, \ldots, N, \; N = 20$

You can reduce the dimension by extracting factors that explains the variation in X_t vector

The vector of factor is limited.

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r = 1 . . . 10

COMMON FACTOR SPACE

TRUE FACTOR SPACE

you have to club the information in 5, 10, or 2 series. This is to reduce the dimension.

Based on heterodox you use factor analysis for policy analysis and forecasting exercise.

$X_t = (x_{it}, \ldots, x_{Nt})$

↳ vector of macroeconomic variables

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$i = 1 \ldots N$ ← CROSS SECTIONAL DIMENSION

$t = 1 \ldots T$ ← TIME SERIES DIMENSION

GDP INF MO M1 M2 . . . uncapfg . . .

T

1960

2015

MODEL

$X_t = \Lambda_t F_t + e_t$

X_t: T x N (T ↓ rows, N → columns)

Λ_t: matrix of factor loadings

F_t: vector of factors

e_t: idiosyncratic error terms

e_t is not white noise here because it is not iid

Cross sectional

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It is idiosyncratic if e_t changes with change in X_t

cross is general to specific variables.

capital Λ

$X_t = (\lambda_{1t} \dots \lambda_{Nt})$ matrix of factor loadings.

$F_t = (F_1, \dots, F_N)$ vector of factors

$e_t = e_1 \dots e_N$ ← each cross sectional dependent on error

RESTRICTIONS IMPOSED ON THE MODEL

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1. $\Lambda_t = \Lambda_0$ (constant)
2. F_t and e_t are mutually uncorrelated
3. e_t are serially uncorrelated and i.i.d.
4. e_{it} and e_{jt} are independent for $i \neq j$

(N)

* IF ALL THE FOUR CONDITIONS HOLD THEN THE EQUATION 1 BECOMES EXACT STATIC FACTOR MODEL.

* If e_t are weakly uncorrelated then we have static factor model.

(v) IF CONDITION 3 = FAILS.

i.e.) THE ERRORS ARE WEAKLY CORRELATED

Then, in this case, we have static factor model:

DYNAMIC FACTOR MODEL

$X_t = Z_t$

$F_t = (f_t, f_{t-1}, \ldots f_{t-q})$

$\Lambda_t = \alpha(L)$ LAG OPERATOR

Such that the model becomes

$$Z_t = \alpha(L) f_t + V_t \quad \text{---(ii)}$$

$e_t = V_t$

EQUATION (ii) IS CALLED DYNAMIC FACTOR MODEL.

$$Z_t = \left[I - \alpha_1(L) - \alpha_2(L^2) - \ldots \alpha_q(L^q)\right] f_t + V$$

$$Z_t = \alpha_1 f_{t_1} + \alpha_2 f_{t-1} + \cdots + \alpha_q f_{t-q} + V_t$$

RESTRICTIONS IMPOSED

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1. $X_t = Z_t$

2. $F_t = (f_1', f_2', \ldots, f_{t-q})$

3. $\Lambda_t = (a_0 \; a_1 \; \ldots \; a_q)$

This is called CONTEMPERENOUS DYNAMIC FACTOR MODELLING if all the three case are satisfied.

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If

$X_t = [Z_t, Z_{t-1}, \ldots, Z_{t-p+1}]$

$f_t = (f_t', f_{t-1}', \ldots t_{t-q-p+1})$

$\epsilon_t = (V_t, V_{t-1} \ldots V_{t-p+1})$

$$\Lambda = \begin{bmatrix} a_0 & A_1 \ldots A_p \\ \vdots & \\ 0 & A_b \ldots A_p \end{bmatrix}$$

This is called Full Dynamic Factor Modelling.

DATE

$$y_{t+1} = \beta_t F_t + e_{t+1}$$

$$E(e_{t+1} / x_t, y_t, \beta_t, x_{t-1}, y_{t-1}, \beta_{t-1})$$

$$E(y_{t+1} / x_t, y_t, x_{t-1}, y_{t-1}, \beta_{t-1})$$

POLICY ANALYSIS TO RETRIEVE STRUCTURAL PARAMETER.

$$y_t = \beta_t F_t + e_t$$

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$$X_t = -\Lambda F_t + e_t \quad \text{---} \quad (1)$$

$$X_t = (x_{it}, \ldots, x_{Nt})$$

$i = 1 \ldots\ldots, N = 200$

$t = 1 \ldots\ldots T = \infty$

1) if fixed N and infinity T

2) if fixed T and infinite N

3) if both N and T are large and they tend towards infinity

4) if both N and T are large but $N > T$

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* All the Factors estimated on first 3 conditions have limited asymptotic properties.

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Based on 4th criteria Bai and Ng (2002) shows that factors are extracted based on $N > T$ the factors will still have some asymptotic properties like normal N regressors

we prefer the cross sectional dimension to be larger than time series dimension

$$Y_t = \beta F_t + e_t$$

$$Y_t = X_1 \ldots\ldots X_{20}$$

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PAGE

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HOW DO WE EXTRACT FACTORS

1. PRINCIPAL COMPONENT ANALYSIS (Pure statistical procedure)

2. BAYESIAN LIKELIHOOD

Bernake and Boivain (2002)

Bernake, Boivain and ELiaz (2005)

IMPORTANT

X_t = GDP Inf u i exg ex Rx

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① Kalman and Caster (1994)

② Otrok and Whitman (1998)

PCA

Components	Eigen EV value	Variation	Cumulative Variation
1	$X_1 = 3$	0.80	0.80
2	$X_2 = 1$	0.10	0.90
	$1 = 0.9$	0.02	0.92
	$X_7 = 0.0$	0.006	

it means 80% of the variation is explained by 1st comp.

Common factors = X

True factor = 7

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It has to be → PAGE 00

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Select the components

EV ≥ 1 (It is not sacrosanct)

In the last example 2 components qualify

So, in Y estimation will only include F_1 and F_2

F_1 (GDP) and F_2 (Inf) do not know if any one of the 7 it is.

$Y_t = \beta_1 F_1 + \beta_2 F_2$

observed

* Then we have something called common factor space defined by q

$q = (F_1 \cdots F_7)$

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PAGE

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True factor space = p

$p = (F_1, F_2)$

EV ≥ 1 (we can look from screw plot and decide to drop)

WE HAVE TO DETERMINE EIGEN VECTOR

EV ⇒ Components loadings.

Variables	1	2	. . .	6	7
GDP	0.9	-0.01	0.1	. .	. .
Inf	-0.1	0.8	. . .	.	.
u	0.75	0.2	. . .		
i	0.1	0.6			
⋮					
RX					

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PAGE

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The loadings will be less than 1

All real variables have a good loading in first components

In the second the nominal variables have a good loading

So now, the first and second component will serve as factors

The first component will be called as Economic activity. The second will

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PAGE

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be called as money demand.

This is Λ (Factor loading)

$$X_t = [\lambda_1, \lambda_2] \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Fitting the above in Y_t

$$Y_t = \beta_1 EA + \beta_2 mq$$

λ_1 = 1st Colourmn entirety

λ_2 = 2nd Colourmn entirety.

In the last table, it is the n dimension that matters. and not the time series. data.

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H means you are summarizing a large component of economic variable.

HOW DO WE DETERMINE COMMON FACTOR SPACE FROM TRUE FACTORS SPACE

1.

$$Y_t = \beta_1 F_1$$

$$Y_t = \beta_1 F_1 + \beta_2 F_2$$

$$Y_t = \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3$$

$$Y_t = \beta_1 F_1 + \beta_2 F_2 + \cdots + \beta_7 F_7$$

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PAGE

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Take the model with least value of information criterion (BIC, AIC, ...)

$q \neq r$

2. → Phillips curve.

$$\pi_t = \beta E(\pi_{t+1}) + K_1 y$$

F_1 F_2

The theoritical model will also inform you the number of true factors.

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(3) Bai and Ng (2002) came up with another ICs (Information criterion) to select the number of factors.

(4) Onatski (2006 b) — base slope of the scree plot.

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PAGE

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MODELLING AND FACTORS

Why do we need factors in economic modelling?

Most of theorifical concepts in economics have no empirical measurement.

For instance, the concept of economic activity, GDP is represented in eco. literature. But there are different things apart from GDP.

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PAGE

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b. Some variables will be multiple

For example, inflation is represented by different components of Inflation like,

CPI, Rural - CPI

c. Policy Perspective

(a) central bank

(b) econometric modeller

CB have their target of

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inflation

FOMC — observed all macroeconomic data

EM — only observe a fraction of data

FOMC and EM observe only fraction of the data

y_t = vector observe variable

Consider 2nd and 3rd case

unobservable — vector of information

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$X_t = [X_{1t} \quad \dots \quad X_{nt}]$

X_t = [all agg of variables]

They have another vector to target

we have to use X_t to extract F_t

F_t = unobserved factors

EXAMPLE (TAYLOR)

$i = \alpha_1 \Pi_{t-1} + \alpha_2 y_{t-1} + e_t$

i = purely observable so belongs to y vector

Π = " " " " " " But this is not explicitly stated

If it is explicit then it is X_t vector

y_t is not observable. It has to be observed from X_t vector

$X_t = [X_{1t} \quad \dots \quad X_{nt}]$

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all macroeconomic variables

guide them to achieve the

$X_t = \Pi F_t + e_t$

cannot be properly estimated. from large macroeconomic variables,

PRIZE PUZZLE is not observable when the inflation is not observable. Therefore, we have to follow the same process like that of y_t

F_t = [factor of EA is extracted from X_t]

DSGE MODEL

$$\pi_t = \alpha_1 + \alpha \pi_{t-1} + \alpha_2 E(\pi_{t+1}) + \lambda \bar{y}$$

$$y_t = \beta_1 + \beta_2 E(\hat{y}) - \beta_3 [i - E(\pi_{t+1})]$$

Closure rule

$$i = \gamma_1 + \gamma_2 \pi_t E(\pi_{t+1}) + \gamma_3 \bar{y}$$

NEW KEYNESIAN DSGE MODEL.

X_t = [All macro economic variables]

This is given in the model.

(i) We have different inflation series

(ii) EA represented by $\bar{y}$

(iii) Real rate are unobserved. i is observed.

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$Y_t = [i]$

$X_t = \Lambda \cdot F_t$

$F_t (F_1 - - - - F_w)$

$F_t = [F_\pi, F_{EA}, F_{Ri}]$

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PAGE

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(S)FA-SVAR

(S)(FA)

Structured because Λ will be a diagonal matrix in $X_t = \Lambda F_t$

The inflation and EA can load in one factor in one factor. Then which ones does it specify it difficult to conceptualize.

The above is the problem in PCA.

The problem becomes cumbersome in Bayesian likelihood.

If Λ is diagonal matrix then X_t also becomes a diagonal matrix.

$$\begin{bmatrix} X^Y \\ X^\Pi \\ X^i \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & & \\ & \Lambda_{22} & \\ & & \Lambda_{33} \end{bmatrix} \begin{bmatrix} F^Y \\ F^\Pi \\ F^i \end{bmatrix}$$

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The above is called

S-FSM (Structural - Factor Static Model)
S-FDM (" " Dynamic Model)

According to Bai and Ng the requirement is to have large set of macro economic variable of MT, where MT.

THE MODEL

$$\begin{bmatrix} y_t \\ F_t \end{bmatrix} = \Gamma_1 \begin{bmatrix} y_{t-1} \\ F_{t-1} \end{bmatrix} + \cdots\cdots + \Gamma_p \begin{bmatrix} y_{t-p} \\ F_{t-p} \end{bmatrix} + Be_t$$

Factor is augmented in the SVAR and it becomes FA-SVAR.