LUPTAKOHL (2004)

H_0: NO C.I. IN PRESENCE OF ONE POINT SHIFT at AT deterministic component (prefarbly at level) KNOWN DATES

H_1: ~~NO~~ C.I.

in order to study the unification of Germay in 1990. ~~not~~ the break dates a priori.

They introduced two differen

1) Impulse dummies d_{0t},

2) Step dummy, d_{01}

classmate

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suppose break at 1979 (83) so only that value is

0 1 0 0 0

actual break date

$$d_{0t} = \begin{cases} 1 & \text{if } t = T_0 \\ 0 & \text{if } t \neq T_0 \end{cases}$$

$t = 1, \ldots, T$ (sample size)

break dates $\to T_0, T_1 < T$

$$d_{1t} = \begin{cases} 0 & \text{if } t < T_1 \\ 1 & \text{if } t \geq T_1 \end{cases}$$

$T_0 < T$

1990 — 2000

exceptional case but can happen

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$1 < T$; $\boxed{T_0 = T_1 = T}$

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Step Dummy

Unification happens at 1990

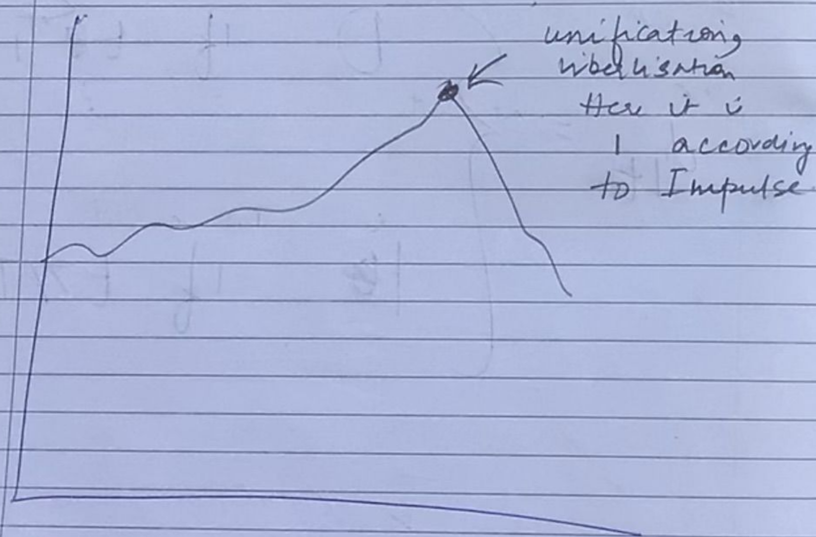
to all values after that

will take 1

0 0 1 1 1 1 1 1

THIS IS THE NORMAL

WAY OF TAKING DUMMIES



DATA GENERATING PROCESS

level, trend, Imp. Dummy

$$y_t = u_1 + u_2 t + \delta_{0t} + \delta_{1t} + x_t$$

step dummy

$$y_t = [y_{1t}, y_{2t} \cdots\cdots y_{nt}] \sim I(1)$$

vector

$$x_t \sim \text{i.i.d}[0, \omega]$$

omega is unknown.

The wanted to see how

the dummy wanted

to affect level and

trend.

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$$X_t = A_1 x_{t-1} + A_2 X_{t-2} + \cdots$$

$$\cdots + A_n X_{t-p} + e_t$$

Vector Auto Regression of errors

Substract X_{t-1}

we have $X_t - X_{t-1} = DX_t$

$$DX_t = \sum_{n=1} A_n D X_{t-p} + e_t$$

VECM REPRESENTATION

$$DX_t = u_1 + u_2 t + \underline{\Pi X_{t-1}} + \sum_{n=1}^{t-1} A_n D X_{t-p} + e_t$$

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These dummies will affect the level

In this Π is full rank.

X_{t-1} is vector of errors.

$$X_t = y_t - u_1 - u_2 - \delta_{0t} - \delta_{1t}$$

Collapses to Joh. (1776)

long-run

$\Pi = \alpha \beta'$

Π has full rank

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HYPOTHESIS

Rank of Π

$H_0(r_0)$: $rk(\Pi) = r_0$

r_0 = assumed rank

$H_1(r_1) = rk(\Pi) > r_0$

Examp:

$r_0 = 0$

$r(k(\Pi)) = 0$ NO C-I..

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How do we estimate this eq.

$$y_t = u_1 + u_2 t + \delta_{0t} + \delta_{1t} + x_t$$

We will apply Generalized least square to derive this.

(GLS) ← Look for properties

we cannot use OLS because

1990 ≐ H_0 : NO CI PRESE-NCE OF 2 ST

	d_{01}	d_{02}	d_{1t}	d_{2t}
	0	0	0	0
	0	0	0	0
	0	0	1	0
1990	1	0	1	0
	0	0	1	0
	0	0	1	1
2008	0	1		0 1
	0	0		0 1
	0	0		0 1

Important to ~~[illegible]~~ assume the break date are independent

WE CAN BUILD A

MODEL FOR h

$$y_t = u_1 + u_2 t + \sum_{t=1}^{i=1 \ldots n} m \, d_{oit} + \sum_{j=1 \ldots z}^{t=1} + X_t$$

Another specification

is $u_2 = 0$

LUTKEPOHL, SAIKONNEN AND TRENKLEU (2004)

This is yesterday's

H_0: NO C.I, WITH A SHIFT AT KNOWN DATE

H_1: ~~$H_1(r_0)$~~: NO C.I

$$H_0(r_0) : rk(\Pi) = r_0$$

$$H_1(r_0) : rk(\Pi) > r_0$$

H_0: NO C.I. WITH SHIFT AT UNKNOWN BREAK DATE

H_1: NO. C. I

The difference is in the case the shift ~~is~~ happening at unknown dates

The break dates are endogenous

DGP

$$y_t = u_1 + u_2 t + \delta_1 \tau + \gamma_t$$

where τ denotes the break date (that is, where the shift occurs and is known)

Defination of Dummy

The previous dummy in the previous ~~???~~ dummy will not work.

- τ is fixed fraction of the sample

$t = 1, \ldots, \tau, \ldots T$.
tau

$$d_{1t,\tau} = \begin{cases} 1 & \text{if } t \geq \tau \\ 0 & \text{if } t < \tau \end{cases}$$

similar to step dummy

τ is unknown

So, we have to define τ.

$\tau = [\cdot]$

$\tau = [T\lambda]$

where $0 < \underline{\lambda} \leq \lambda \leq \bar{\lambda} < 1$

where $\underline{\lambda}$ and $\bar{\lambda}$ are integers which can be chosen arbitrary but

$\underline{\lambda} \longrightarrow$ lower limit

$\bar{\lambda} \longrightarrow 1$ upper limit

$0.02 \leq \lambda \leq 0.8$ (Example)

MODEL

$$y_t = u_1 + u_2 t + \delta_{1t,\tau} + x_t$$ — (DGP)

$$y_t = [y_{1t}, y_{2t}, \ldots y_{nt}]$$

$$y_t = \underbrace{u_1 + u_2 + \delta_{1t,\tau}}_{\text{Deterministic component}} + \underbrace{A_1 y_{t-1} + \ldots + A_n y_{t-1}}_{\text{level VAR component}} + \underbrace{x_t}_{\text{error}}$$

VAR $(X)-p$

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots\cdots + \alpha_n X_{t-p} + e_t$$

VECM REPRESENTATION (Substracting by X_{t-1})

$$\Delta X_{t-1} = \Pi X_{t-1} + \sum_{i=1}^{t-1} \Gamma_i \, D X_{t-1} + e_t$$

$\Pi = \alpha \beta'$, full rank matrix

Null Hypothesis : $r_0 = 0$ No C.I. in presence of

<u>Estimators</u> ML , LR

Maximum Likelihood
Likelihood Estimators

The critical values are not immune to break date we can include impulse dummy and can include many breaks

$$y_t = u_1 + u_2 t + \delta_i t_1 \tau + \delta_j t_1 \tau + A_1 y_{t-1} + \dots + A_n y_{t-p} + x_t$$

impulse dummy

$i = 1 \dots x$ $\quad 0i = 0$

$j = 1, \dots x$ $\quad y = 1$

DATE 10 03 2016

INOUE (1999)

paper was submit for publication in 95

alternative version of Gregory and Hansen.

a) Residual based.

This is not Residual based.

H_0: NO C.I In the presence of a break in level and/or trend happening at one unknown

H_1: C.I.

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b) In this the shift only affects deterministic component and no regime shift like G&H

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Data Generating Process (The break dates are unknown here.. Data dependent endogen-ous here)

$$y_t^i(\lambda_0) = \sum_{j=1}^{p} \theta_j y_{t-j}^i + e_t \quad \text{for} \quad i = A, B$$

$$y_t^A(\lambda_0) = \sum_{j=1}^{p} \theta_j \, y_{t-j}^A + e_j \quad \text{—— MODEL A}$$

$$y_t^B(\lambda_0) = \sum_{j=1}^{p} \theta_j \, y_{t-j}^B + e_j \quad \text{—— MODEL B}$$

where, $y_t^A(\lambda_0) = X_t - u - \hat{u} D u_t(\alpha)$

$$y_t^B(\lambda_0) = X_t - \mu - \hat{\mu} D u_t(\alpha) - \delta t - \delta \wedge I_t(\lambda)$$

Latin variable.

$$X_t - \mu - \hat{\mu} D u_t(\alpha) = \sum_{j=1}^{p} \theta_j y^{\wedge}_{t-j} + e_t$$

$$X_t = \mu + \hat{\mu} D u_t(\alpha) + \sum_{j=1}^{p} \theta_j X^{\wedge}_{t-j} + \varepsilon_t.$$

in VAR form

Vector of observed time series

Dumm variable that affect level of the.

This is model A

Only the level is affected.

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For B model — both the level and trend is affected.

$$X_t - \mu - \tilde{\mu}\, DU_t(k) - \delta_1 t - \delta_2\, \Delta T_t(k)$$

$$= \sum_{j=1}^{p} \theta_j\, y^{B}_{t-j} + e_{2t}$$

$$X_t = \mu + \hat{\mu}\, DU_t(k) + \delta_1 t + \delta_2\, DT_t(\alpha)$$

$$+ \sum_{j=1}^{p} \theta_j\, X_{t-1} + e_t$$

For C Model

$$y_t^c = c + \hat{\mu}\, D u_t(\lambda) + \sum_{j=1}^{p} \theta_j \cdot y_{t-j}^c + e_{t3}$$

$$y_t^c = x_t$$

$$x_t = \boxed{c} + \hat{\mu}\, D u_t(\alpha) + \sum_{j=1}^{p} \theta_j \cdot x_{t-j} + e_{t3}$$

$$y_t^A = X_t - \hat{\mu} - \hat{\mu} \, DU_t(\lambda)$$

$$X_t = y_t^A + \mu + \hat{\mu} \, \Delta u_t(\lambda)$$

$$X_{t-1} = y_{t-1}^A + \mu + \hat{\mu} \, D(u_{t-1})(\alpha)$$

DEFINATION OF DUMMY VARIABLE

$$DU_t(\lambda) = \begin{cases} 1 & \text{if } (t > [T\lambda]) \\ 0 & \text{otherwise} \end{cases}$$

$$DT_t = \begin{cases} 1 & \text{if } (t - [T\lambda])(t > [\lambda T]) \\ 0 & \text{otherwise} \end{cases}$$

where $I(\cdot)$ is an indicative function.

$\lambda \in \beta$, which is the break function and it has the close set of $[0, 1]$

CO-INTEGRATING FRAMEWORK

$$Dy_t^A(\lambda) = \Pi y_{t-1}^A(\lambda) + \sum_{j=1} \Gamma_j \, y_{t-j}^A(\lambda) + u_{t_1}$$

VECM SPECIFICATION OF MODEL A.

$$Dy_t^B(\lambda) = \Pi y_{t-1}^\beta(\lambda) + \sum_{j=1} \Gamma_j \, Dy_{t-j}^\beta(\lambda) + u_{t_2}$$

VECM SPECIFICATION OF MODEL B.

$$D y_t^c(k) = c + \hat{\mu} D u_t(x) + \Pi y_{t-1}^c(x) + \sum_{j=1} D y_{t-j}^c(k) + e_t$$

EXAMINING THE Π matrix

$\Pi = \alpha \beta'$ $\beta' X_{t-1}$ — C.I VECTOR

concept of rank

$1 \leq q \leq n$ — total no. of variables in X vector.

$\alpha \beta' = n \times q$

rank

$0 \leq r \leq q$

HYPOTHESIS

H_0 : $rk(\alpha) = rk(\beta) \leq r$

r is the full rank of the matrix

H_{1a} : $rk(\alpha) + rk(\beta) = r+1$

H_{2a} : $rk(\alpha) = rk(\beta) > r$

PRACTICALS

$$y_t = \mu + \hat{\theta}^A DU_t + \hat{\beta}^A t + \hat{d}^A D(TB)_t + \hat{\alpha}^A y_{t-1} + \sum_{i=1}^{k} \Gamma_i \, Dy_{t-1} + \epsilon_t$$

insignificant

H_0: $\hat{\alpha}^A = 1$, $\hat{\beta}^A = 0$, $\hat{\theta}^A \simeq 0$, $\hat{d}^A \simeq 0$

H_1: $\underline{\underline{\hat{\alpha}^A}} < 1$ $\hat{\beta}^A$, $\hat{\theta}^A \neq 0$ $\hat{d}^A$ = significant

coefficient lagged

of $DU_t = \begin{cases} 1 & t \geq T_B \\ 0 & \text{otherwise} \end{cases}$

$D(TB_t) = \begin{cases} t & t > T_B \\ 0 & \text{othe} \end{cases}$

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MODEL B

$$y_t^B = \hat{\mu}^B + \hat{\theta}^B DU_t + \hat{\beta}^B t + \hat{\gamma}^B DT_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} r_i D'_{t-i} + e_t$$

H_0: $\hat{\alpha}^B = 1$, $\hat{\gamma}^B = 0$, $\hat{\beta}^B = 0$ $\hat{\theta}^B \simeq 0$

H_1: $\hat{\alpha}^B < 1$ $\hat{\gamma}^B \neq 0$ $\hat{\beta}^B \neq 0$ $\hat{\theta}^B =$ significant

$$DT^* = \begin{cases} t - T_B & 1 \text{ if } t > T_B \\ & 0 \text{ otherwise} \end{cases}$$

This is adjusted version of ADF test

MODEL C

$$y_t = \hat{\mu}^c + \hat{\theta}^c DU_t + \beta^c t + \gamma DT_t + \hat{d}^c D(TB)_t + \hat{\alpha}^c y_{t-1} + \sum_{i=1}^{k} \Gamma_i D y_{t-i} + e_t$$

H_0: $\hat{\alpha}^c = 1$ $\gamma^c = 0$, $\beta^c = 0$ $d^c \simeq 0$

H_1: $\hat{\alpha}^c < 1$ $\gamma^c \neq 0$ $\beta^c \neq 0$ $\hat{d}^c \simeq$ significant