DATE 01 03 2016

# CO-INTEGRATION WITH BREAKS

$H_0$ : NO C-I

$H_1$ : C.I

1. CDOW (1987)
2. EG ← Residual
3. Johanssen.
4. P,SS (1999, 2001)
5. Narayan (2005)

All these tests no breaks or regime shifts in co-integration vectors

The second type of tests in where

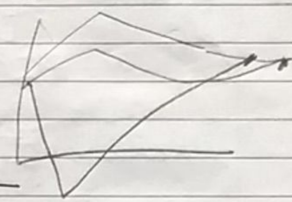
a) breaks can exist





and / or $\alpha_1 y_t + \alpha_2 x_t$.

b) Regime shifts



## TYPES OF TESTS

1) GREGORY AND HANSEN (1996 a, 1996 b)

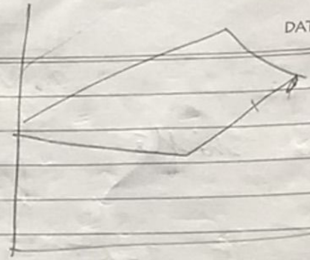
2) JOHANSEN ETAL (2000)

3) Carrion-I-Sylvestre and Sanso (2006)

4) Luptaphol et al (2004)

5) Salkkanan and Luptaphol (2009)

6) Inoue (1999)



Existence of break does not mean regime shift

~~we~~ ~~conn~~ The conclusion will change of the hypothesis will change

GREGORY AND HANSEN (1996a, b)

developed Residual based test

for

$H_0$: ND C.I.

$H_1$: C.I.

with

a) ~~trend shift~~ No break or regime shift

b) level shift





c) level with trend

d) Regime switching [shift of level and slope]

There are four alternatives here

$$y_t = \alpha_1 + \alpha_2 X_t + e_t$$

$$e_t = Y_t - \alpha_1 - \alpha_2 X_t$$

$$e_t = \gamma e_{t-1} + \sum_{i=1} \Delta B_i e_{t-1} + w$$

This works for the first alternative; This is just repitition of EG.

Therefore, this model (alternative) becomes the null hypothesis

$H_0$ : $Y_t = \mu_1 + \phi^T X_t + e_t$

$X_t$ = vector of exogenous variable with $I(1)$ process

$X_t \sim I(1)$ $\quad e_t \sim I(0)$

This null to be tested against three alternatives.

$H_1$ : C.I with unknown single break date at level

: with level shift at unknown date (endogenous break date)

(multivariate version of Zivoth andrews)



If this is know then it is multivariate version of

The dummy has to be defined

$$\gamma_{t\tau} = \begin{cases} 0 & \text{if } t \leq [n,\tau], \ t \in T \\ 1 & \text{if } t > [n,\tau] \end{cases}$$

(gamma)

unknown, tau (endogenous)

Scaling parameter

This is the dummy variable.

$$y_t = \mu_1 + \mu_2 \gamma_{t\tau} + \Phi^T x_t + z_t$$

$\mu_1$ = is the level of the relation before the shift

$\mu_2$ = is the level of the relation after the shift.

## MODEL 3 [level shift with trend]

MODEL [C/T] just add the trend.

ALTERNATIVE

$H_{1C}$ :

$$y_t = \mu_1 + \mu_2 \Upsilon_{\tau} + \underbrace{\beta t}_{\text{trend}} + \overbrace{\Phi^T}^{\text{Matrix}} \underbrace{X_t}_{\text{vector}} + e_t$$

IT MEANS

$H_{1C}$ : C.I WITH IN PRESENCE OF LEVEL SHIFT AND TREND

MODEL [C/S]

## MODEL 4 [REGIME SHIF]

Alternative

$$H_{1d} : y_t = \mu_1 + \mu_2 \Upsilon_{1\tau} + \Phi_1^T X_t + \Phi_2 \Upsilon_{2\tau} X_t + e_t$$

$\Phi_1$ = parameter before regime shift (R.S)

$\Phi_2$ = parameter after R.S

imp Both the slope and level will hav "Regime"

$\mu_1$ = level parameter befor R.S.

$\mu_2$ = " " after R.S.

The 3 test statistic developed

1) ADF
2) $Z_\alpha$ ← just like Philips and Perron.
3) $Z_t$

He realized that, in the presence of breaks will lead to false conclusion

This was published later onward.

GH (1996 b)

Here, a more comprehensive model (where regime shift and trend shift)

MODEL [C/S/T]

$H_0$: NO C.I. [NULL SAME]

$H_{1e}$: C.I. in presence of regime shift and trend shift at a unknown date

such that his model is nothing but

trend also has a dummy concept

$$y_t = \mu_1 + \mu_2 \tau_{1\tau} + \beta t + \beta_2 t\, \tau_{2t} + \phi_1^T x_t + \phi_2^T x_t \tau_3 + e_t$$

$\mu_1, \beta_1, \theta_1$ parameter before shift

$\mu_2, \beta_2, \theta_2$ " after "

$n = 300$ (sample)

10,000 - iterations

no convergence for one week.

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Johansen S. Mosconi R. and

Nielson B (2001)

Peron (81) one break

Johansen (2000) — multiple breaks

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Johanssen's

$$\Delta w_t = \Pi w_{t-1} + u + e_t + \sum_{i=1}^{p-1} D_i w_{t-p}$$

$$\Delta u_t = u + \delta t + \sum_{i=1}^{p-1} D_i X_{t-p} + \alpha_i \begin{bmatrix} w_t \\ 1 \\ \delta \end{bmatrix}$$

assuming $\Pi$ matrix $\begin{bmatrix} \beta & u_2 & t \end{bmatrix}$

$+ e_t$

$$\Delta u_t = \Pi w_{t-1} + \Pi_1 t + u + e_t$$

$$= \Pi = \alpha \beta'$$

$$\Pi_1 = \alpha \gamma'$$

Co-integrating vector

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$$DW_t = \alpha \beta' w_{t-1} + \alpha \gamma' t$$

$$+ \mu + e_t$$

$$DW_t = \underset{\substack{\uparrow \\ \text{Coefficient of speed of adj.}}}{\alpha} \underbrace{\left(\beta' w_{t-1} + \gamma' t\right)}_{\text{This is the form of the co-integrating vector}} + \mu + e_t \quad (2)$$

This is $H_1(r)$ model allows for co-integration when there is linear trend in the model.



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In pressaine sche

$\beta'$ – the rank is $n-1$

Here it should be full rank

RESTRICTED VERSION

First case

IF $\gamma' = 0$

$$DW_t = \alpha \left(\beta' w_{t-1}\right) + \mu + e_t$$

speed of adj.

only imp. in S.R. still there is a trend when $\gamma$.

However, when we meet the Trend has died

This is called $H_{1C}(r)$ model.

Although the Trend is not there in C.I. vector

## Second Case

$\gamma = 0$ $\quad \mu = \alpha p'$ $\quad$ no break in level and trend

$$Dw_t = \alpha(\beta' w_{t-1}) + \alpha p' + e_t$$

$$Dw_t = \underbrace{\alpha(\beta' w_{t-1} + p')}_{} + e_t$$

break in level.

There is no trend here

because the factorising $\alpha$ is not of $\gamma t$ but of $\alpha p'$

$H_c(r)$ model



## GENERAL SETTING OF MODEL

$$Dw_t = [\Pi_1 \;\; \Pi_t] \begin{bmatrix} w_{t-1} \\ t \end{bmatrix} + \sum_{i=1}^{p=1} \Gamma_i \, Dw_{t-p} + e_t$$

$w_t$ = vector of macro economic variable

$w_t = (X_t, Y_t, Z_t)$

and assume multiple break dates

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Divide $X_t$ vector $t_1 = 1 \ldots q$

into two $t_2 = q+1, \ldots n$

Samples

$\Pi_0 \, w_{t-1} + \Pi_i \, t$

role of trend

$i$ = no of break

$DW_l = [$

DEFINE THE $H_l(r)$ HYPOTHESIS

$$H_L(r) = \begin{bmatrix} \Pi_0 & \Pi_i \end{bmatrix} \begin{bmatrix} w_{t-1} \\ t \end{bmatrix} \leq r$$

OR

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$$H_L(R) : \begin{bmatrix} \Pi_0 & \Pi_i \end{bmatrix} \begin{bmatrix} w_{t-1} \\ t \end{bmatrix}$$

$$= \alpha \begin{bmatrix} \beta' \\ \gamma_1' \\ \vdots \\ \gamma_i' \end{bmatrix}$$

breaks

break at the slope of trend function

The second matrix is changed. The break happens at trend.

$\alpha \beta' = \Pi_i w_{t-1}$

$\Pi_i t = \alpha \gamma_1' + \alpha \gamma_2'$

(1) — $H_L(r) : [\Pi, \Pi_i] \leq r$

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EXPANDING.

(2)

rank of specific matrix

$H_L(r)$ : rank $[\Pi_1 \quad \Pi_{21} \quad \Pi_2 \quad \dots \quad \Pi_i] \leq r$

1st sample $- \Pi_1 . \Pi_i$

2nd $\Pi_2 \; \Pi_i$

when $\gamma = 0$

then

$[\Pi] \leq r$ because — Co-integration if the inequality is positive

$[\Pi_1 \quad \Pi_2 \quad \dots \quad \Pi_i] = 0$

(3) — $H_{Lc}(r)$ : $[\Pi] \leq r$ — same as Johansen with no break (1986) paper

Finally

(4) — $H_c(r)$ : $[\Pi_1 \quad u_1 \quad \dots \quad u_i] \leq r$

The alternative is to introduce some Dummy into the variable.

$$D_{i,t} = \begin{cases} 1 & \text{for } t = T_j - 1 \\ 0 & \text{otherwise} \end{cases}$$

it should some sample because can have my sample

summation of Dummy's

$$E_{j,t} = \sum_{j=K+1} \underbrace{D_{j,\,t-1}}_{\text{observation in each sample}} = \begin{cases} 1 & \text{for } T_{j-1} + K + 1 \le t \le T_j \\ 0 & \text{otherwise} \end{cases}$$





$$t = j - 1$$

$$j = \underbrace{2, \ldots, q-1}_{K}$$

vector of breaks

$$E_t = [E_{1t}, E_{2t}, \ldots \ldots E_{q,t}]$$

BASED ON ABOVE, THE MODEL

L.R. Component

$$DW_t = \alpha \begin{bmatrix} \beta' \\ \gamma' \end{bmatrix} [W_{t-1} \quad \underbrace{t E_t}_{\text{multiplication Dummy}}] + u E_Z + \sum_{i=1}^{k-1} \Gamma_i DW_{t-1}$$

$$\sum_{i=1}^{K} \sum_{j=2}^{q} K_{i,j} D_{j,t-i}$$

$$+ e_t$$

From this we can construct L.R. Hypo

$$H_{L(r)} = [\Pi, \alpha\gamma' E_t] \leq r$$

K, q different samples

$$H_{LC}(r) = \Pi \leq r$$

$$H_{L(r)} = [\Pi, u E_t] \leq r$$

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## CARRION - I -

## SYLVESTRE AND SANSO (2006)

allows for one break in co-int. that happens in null

$H_0$ : C.I with one break.

(In (Gregory, Hansen (it is no co-int. (and $H_1$ : co-int.)

In this they came up with C.V of

- Known
- Unknown.

$H_1$ : NO C.I.

This is LM-based based while Greg-Han is Residual based test.



Deterministic tr

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DGP

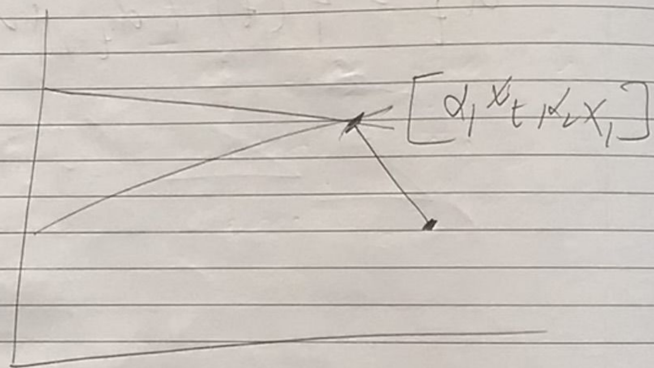
$$y_t = \alpha_t + \gamma_t t + \underbrace{x_t' \beta}_{\text{C.I. Vector}} + e_t$$

$$x_t' = x_{t-1} + v_1$$

$$\alpha_t = f(t) + \alpha_{t-1} + v_2.$$

$$Dy_t = \underbrace{\underbrace{\mu_1 + \delta t}_{\text{Determ}} + \underbrace{\sum_{i=1}^{p-1} \Delta\Gamma_i y_{t-p}}_{\text{Stoch}}}_{\text{VAR}} + \underbrace{\mu_2 + \Pi y_{t-1} + \delta_2 t}_{\text{c.v.}} + e_t$$

Stochastic




The co-integrating vector is also changing

So, we have stochastic component is also changing.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$x_t \sim I(1)$

$v_2 \sim i.i.d(0, \sigma_n^2)$

$$\alpha_t = \underbrace{f(t)}_{1} + \alpha_{t-1} + v_2$$

This is a function indicating level change.

Dummy

$$DT_b = \begin{cases} 1 & \text{for } t = T_b + 1 \\ 0 & \text{otherwise} \end{cases}$$

break date

Similar ok and papers test

$$DU_t = \begin{cases} 1 & \text{for } t > T_b \\ 0 & \text{for otherw.} \end{cases}$$

This is the known and unknown here. ~~Ba~~ based on ~~w~~ if you give the break or the the data take out the break

~~FIRST MODEL~~

~~$Y = 0 \; 2 \; f(t) = \theta D T_b$~~

COMPLETE TE

$$y_t = f(t) + \alpha_{t-1} + v_2 + \gamma_t + \beta(x_{t-1} + v_1) + \epsilon_t$$

$$= f(t) + \alpha_{t-1} + \gamma_t + \beta' x_{t-1} + \underbrace{v_2 + \beta' v_1 + \epsilon_t}_{m}$$

$$y_t = f(t) + \alpha_{t-1} + \gamma_t + \beta' x_{t-1} + m$$

$$m \sim i.i.d\ (0, \sigma_u^2)$$

$$x_{t-1} \sim I(1)$$

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# FIRST MODEL

$\gamma = 0 \quad f(t) = \theta(D\,T_b)_t$

not said anything about that

$Y_t = \alpha_{t-1} + \underbrace{\theta(D\,T_b)_t}_{\text{This break is affecting at one point}} + \beta' x_{t-1} + m$ — A

allows level shift one point.

Further Restrict this model

a) $\gamma \neq 0 \quad f(t) = \theta(D\,T_b)_t$ Deterministic component.

$Y_t = \alpha_{t-1} + \theta(D\,T_b)_t + \gamma_t + \beta' x_{t-1} + m$ — A

level shift with trend

b) If

$\gamma \neq 0 \quad f(t) = \lambda (DU_t)_t$

Model B

$y_t = \alpha_{t-1} + \gamma_t + \lambda (DU_t) t + \beta' X_{t-1} + M$

slope of trend func. is changing

Next

c) $\gamma' \neq 0 \quad f(t) = \theta (DTb)_t + \lambda (DU_t) t$

$y_t = \alpha_{t-1} + \gamma \cdot t + \theta (DTb_t) t + \lambda (DU_t) t + \beta X_{t-1} + m$

Model C : level and slope of trend function to shift.

Hao (1996) —

Bartley et al (2001) — level shift ~~in~~ trend which is model A

It allows for both

deterministic and

stochastic component to

change.

Now, if we assume

multiciplative Dum

$\gamma = 0 \quad f(t) = \theta\,(DT_y)\,t + \beta' X_{t-1}\,(DT_y)\,t$

trend

The model becomes

$$y_t = \alpha_{t-1} + \theta_1 (DT_y)_t + \theta_2 X_t (DT_y)_t + \beta' X_t + e_t$$

affecting on level and C.I. Vector

matrix

$$= \alpha_{t-1} + \theta_1 (DT_b)_t + \underbrace{(\theta_2 DT_b + \beta')}_{\text{slope of the C.I. vector}} X_t + e_t$$

This is the stochastic component

This is Model (D) ← It allows for deterministic and stochastic components to shift (but without trend)

In the next

$$\gamma \neq 0 \quad f(t) = \theta(DT_b)_t + \beta' x_t (DT_b)_t + \lambda(DU)_t$$

Then,

$$y_t = \alpha_{t-1} + \gamma_t + \theta(DT_b)_t + \beta' x_t (DT_b)_t + \lambda(DU_t)_t \cdot \cancel{x_{t-\beta'}} + u_t$$

$$y_t = \left(\alpha_{t-1} + \theta_1 b(T_b)_t\right) + \left(\gamma_t + \lambda(DU_t)_t\right) + \left[\theta_2(DTb)_t + \beta\right] x_t + u_t$$

~~Here~~ both the

trend and deterministic component will change.

Now, set the Hypothesis of the model.

$$y_t = \alpha_t + \gamma_t + \beta' x_t + \epsilon_t$$

$$x_t = x_{t-1} + v_1$$

Restrict the

$$\alpha_t = f(t) + \alpha_{t-1} + v_2.$$

$$x_t \sim I(1)$$

$u_2$.

variance

$H_0: \sigma_u^2 = 0 \leftarrow$ (in favour of C.I.)

$H_1: \sigma_u^2 > 0 \leftarrow$ ( " " NO C.I.)

If $\sigma_u^2 = 0$ (Caveat)

then $\alpha_t = 0 = f(t) + \alpha_{t-1} + \varepsilon$.

Then the model $y_t$ will collapse to.

FOR MODEL A

$y_t = g(t) + \beta' X_t + t$ ; $g(t) = \alpha + \theta(DU_t)t$

where $g(t)$ is an indicative function of whether the break will happen

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$$g(t) = \alpha + \theta(DU_t)_t$$

Null hypothes under $A_2$

$$y_t^{AW} = [\bar{\alpha} + \theta(\underline{DU_t})_t] + \beta' X_t + e_t$$

g(t)

level

dummy variable

FOR DIFF. MODEL ~~B~~ A

$$g(t)_A = \alpha + \gamma t + \theta(DU_t)_t + \gamma_t$$

Dummy affected slope

$$g(t)_B = \alpha + \gamma t + \tau (DT_t)_b$$

Dummy affect level.

$$g(t)_C = \alpha + \theta(DU_t)_t + \tau(DT_b) + \gamma_t$$



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The bone of Contention is g(t)

This is only for deterministic component

AFECTING BOTH DETRMINISTIC AND STOCHASTIC ----

$$y_t = g(t) + X_t \beta_1' + X_t \beta_2' D u_t + e_t$$

$$g_t^D = \alpha + \theta D u_t$$

$$g_t^E = \alpha + \gamma_t + \theta (D u_t)_t + \tau (D T_b)'_t$$